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Traditional and multidimensional poverty measures

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Traditional approach

In the traditional approach every statistical unit (individual or household) is defined as POOR if the corresponding income or total consumption is BELOW a certain threshold defined as POVERTY LINE.

The proportion of poor statistical units (individuals) is defined as *Head Count Ratio* (HCR).

Poverty lines

First classification:

Absolute
Relative

Second classification:

Objective
Subjective

Today we will treat objective P.L., both absolute and relative

Objective poverty lines

- Basic needs criterion
- Food Ratio method
- Percentage of mean or median income (or consumption expenditure)
- Percentile of income distribution (or consumption)
- Official definition
- Other methods

Basic needs criterion

- Rowentree (1901)

b_0= minimum cost on food

$a*b_0$= minimum cost for other basic needs

$$z = b_0 + a * b_0$$

This poverty line is completely ABSOLUTE. It does not depend on average income or consumption in the Country.

Engel method or *Food ratio*

- Two families reach the same welfare level when they show the same food ratio. Higher is the food ratio, poorer is the family.
- Considering the Engel curve:

$$\ln y_0 = \ln(b/y) = \alpha_0 + (\alpha_1 - 1)\ln y$$

- We may get the poverty line:

$$z = \exp\{(\alpha_0 - \ln y_0)/(1 - \alpha_1)\}$$

Two methods that are completely relative are based on income distribution

- Percentage of mean or median income
- Example: International Standard Poverty Line:

  50% of mean

  60% of median (Eurostat definition)

- Exercise: guess why 60%

- Percentile of income distribution

Official definition

World Bank:

Extreme poverty line:
1.25$ per day per person (it was 1$)

Poverty line:
4$ per day per person

Equivalent income or consumption expenditure

In the traditional approach to poverty measurement a monetary variable is taken into account: income or consumption. Total household income or total household consumption are constructed considering all household member. Then for sake of comparison we need to construct the:

- <u>Equivalent income (or consumption)</u>

The total household income is divided by a coefficient (economic index number) defined as: <u>Equivalence scale</u>

Equivalence Scales

- Equivalence scales represent a prerequisite in every study of well-being carried out using measures of income distribution, inequality and poverty; moreover, they constitute a suitable economic tool to incorporate the impact of demographic changes into models of spending allocation for aggregated consumption.
- In fact, households differ in size, composition, and other socio-economic characteristics; this must be considered when our aim is comparing income or total consumption among households.
- Equivalence scales are by one consent considered a necessary tool in poverty analysis, welfare comparison and income distribution analysis; but there is no unanimity in the methodology to be utilised to calculate them.

Equivalence scales

Hagenaars *et al* (EUROSTAT, 1994) suggest the following classification:

- 1. NORMATIVE SCALES
- 2. SOCIAL SECURITY SCALES
- 3. SCALES BASED ON CONSUMPTION
- 3.1. Engel method, only food share is utilised
- 3.2 Budget distribution methods
- 3.3. Utility maximisation
- 3.4. Complete demand systems
- 3.5. Intertemporal equivalence scales
- 4. EQUIVALENCE SCALES BASED ON DIRECT WELFARE MEASUREMENT

Normative Scale

- Normative equivalence scales are based on some norms set by experts in defining a minimum level of consumption or basket of goods for household of different composition and size.

- OECD 70 – 50 SCALE
  - 1 for the first adult
  - 0.7 for each subsequent adult
  - 0.5 for each child under 16

- Modified OECD SCALE EUROSTAT(1994)
  - 1 for the first adult
  - 0.5 for each subsequent adult
  - 0.3 for each child under 16

SOCIAL SECURITY SCALES

- Other sets of scales can be calculated implicitly by social security regulations.
- For example the UK Social Benefit Scale (for family with head below 65 years) is equal to 1 for the first adult, 0.6 for any additional adult and between 0.33 and 0.5 for any child according to age.
- The official equivalence scales for Great Britain (McClements, 1977) are normative scales based on a study conducted with the estimation of a Social Benefit model.

Scales based on consumption expenditure

- 1 Uniequational Models
- 1a. Engel Method, based on share spent on food (food ratio)

- 1b. Rothbarth Method, based on share spent on "adult goods" (i.e. alcoholics and tobacco)

- 2. Complete Demand Systems

- 3. Intertemporal Scales

SCALES BASED ON ENGEL OR FOOD RATIO METHOD

- Engel's method is based on the assumption that household welfare is related with the share of the budget devoted on food.
- Higher is the food ratio, lower is the standard of living.
- The model firstly proposed by Van Ginneken (1982):

$$\log A = a + b \log C + c \log N + u$$

- with constant elasticity:
- the scale can be obtained as follows:

$$\begin{cases} e_1 = 1 \\ e_{n+1} = e_n \left(1 + \frac{\varepsilon}{n}\right) \end{cases} \qquad \varepsilon = \frac{\partial \log C}{\partial \log N} = \frac{1 - c}{b}$$

Poverty indices

- HEAD COUNT RATIO

$$HCR = \frac{1}{n}\sum_{i=1}^{p} 1 = \frac{p}{n}$$

- Measures the diffusion or incidence of poverty, not the intensity.

Poverty indices

- RELATIVE POVERTY GAP

$$RPG = \frac{1}{n}\sum_{i=1}^{p}\left(\frac{y_p - y_i}{y_p}\right)$$

- Measure the poverty intensity, since it takes into account the distance from the P.L.

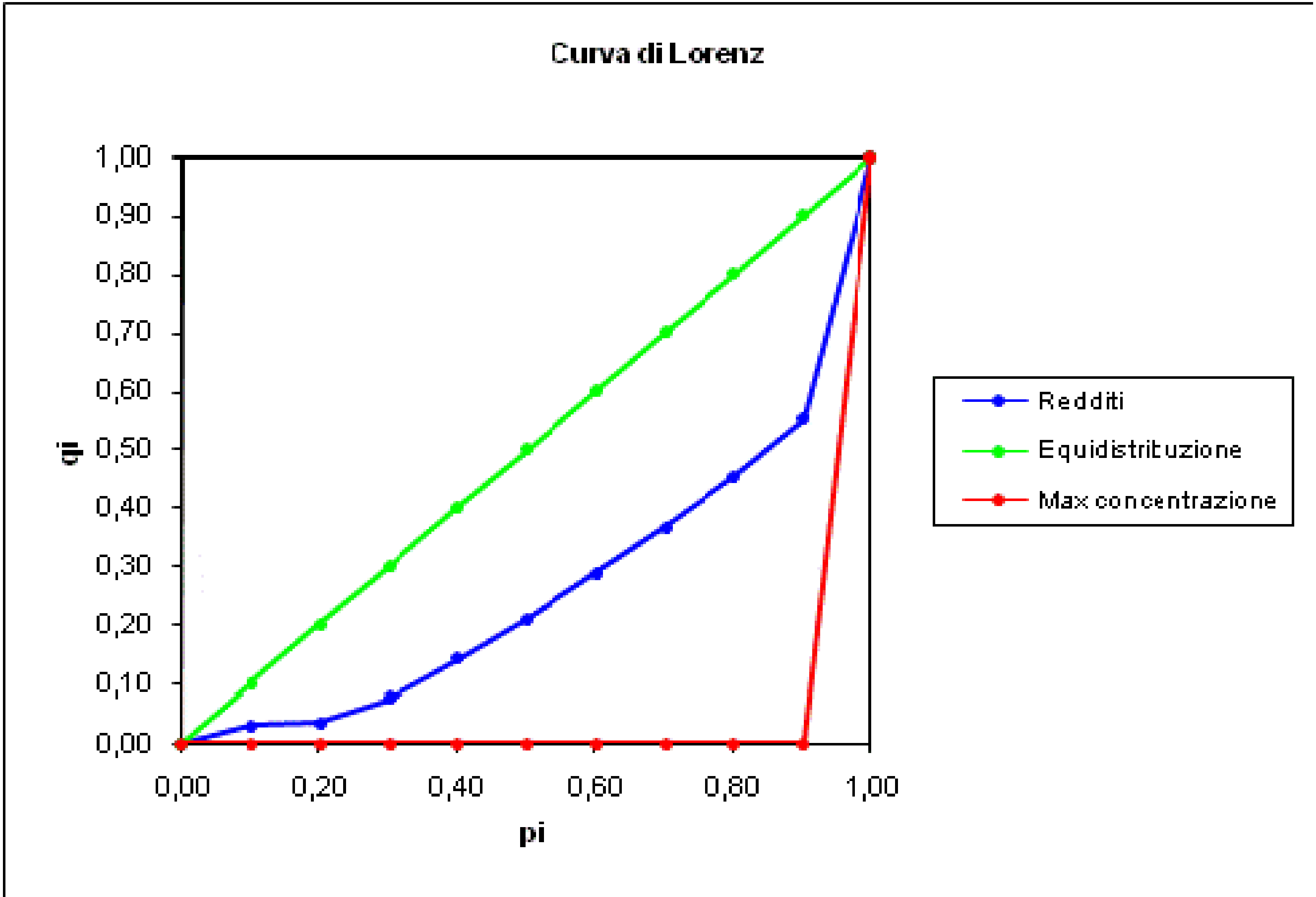
Poverty indices

- GREER-FOSTER-THORBECKE

$$FGT = \frac{1}{n}\sum_{i=1}^{p}\left(\frac{y_p - y_i}{y_p}\right)^{\varepsilon}$$

- For ε =0 is the HCR
- For ε =1 is the RPG
- Per ε =2 is defined as the *Poverty severity index*

Lorenz Curve



Inequality indices

- Gini Coefficient

$$Gini = \frac{2}{n^2} \sum_{i=1}^{n} i . \left(\frac{y_i - \overline{y}}{\overline{y}} \right)$$

- Generalised entrophy indices

$$GE(0) = \frac{1}{n} \sum_{i=1}^{n} - \log \left(\frac{y_i}{\overline{y}} \right)$$

$$GE(1) = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{y_i}{\overline{y}} \right) \log \left(\frac{y_i}{\overline{y}} \right)$$

Sen index (1976)

- Sen (1976) proposed an index combining H, I and the Gini index calculated among the poors:

$$S = H\left\lfloor I + \left(1 - I\right)G_q \right\rfloor$$

- The Sen index could be also seen as a weighted mean of the *poverty gaps,* and re-written as:

$$S = \frac{2}{\left(q+1\right)Nz}\sum_{i=1}^{q}\left(z - y_i\right)\left(q + 1 - i\right)$$

Multidimensional and fuzzy approach

- **Aims of the methodology**

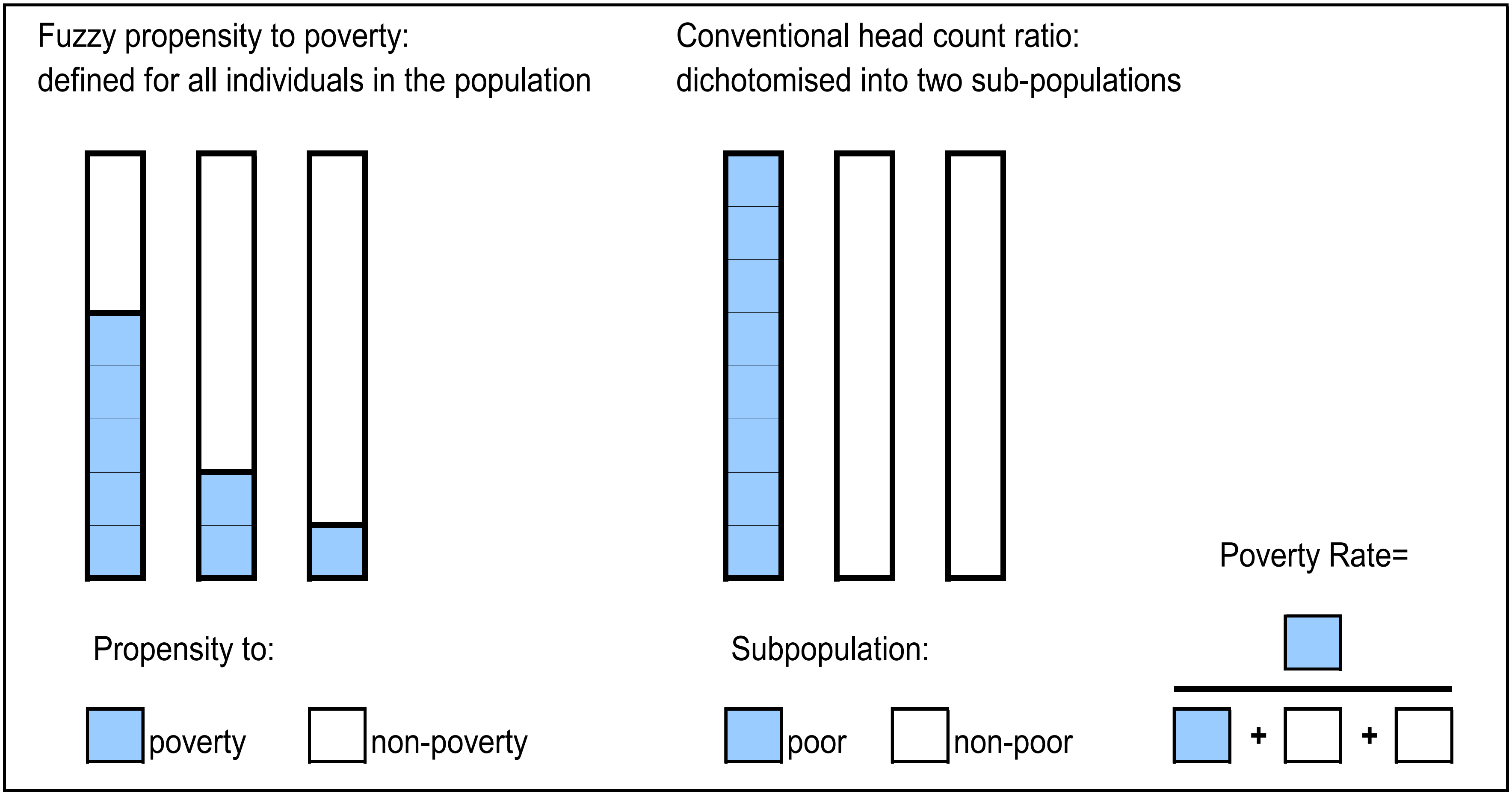
- DEVELOP A SYSTEMATIC APPROACH TO THE EXTENSION OF THE CONVENTIONAL CLASSIFICATION INTO POOR AND NON POOR
- INTRODUCTION OF A MULTIDIMENSIONAL APPROACH BASED ON SUPPLEMENTARY VARIABLE
- DEVELOP A DYNAMIC MODEL ESTIMATING CORRECTLY THE MOBILITY OF INDIVIDUALS.

Multidimensional and fuzzy approach

- **Beyond the income poverty line**

- Poverty as a matter of degree
- Supplementary indicators of the level of living
- Composite supplementary poverty index
- Multi-dimensional poverty: overall index combining income and supplementary poverty
- Poverty dynamics: persistent versus transient poverty

Multidimensional and fuzzy approach



Why treat poverty and deprivation as a matter of degree?

Further insight into the relative income situations of individuals, particularly at the lower end of the income distribution by incorporating a measure of the actual levels of incomes received

Non-monetary deprivation means forced non-access to various facilities/possessions: hence it is inherently a matter of degree

Some *quantitative approach* is essential

Definition of the membership Function based on monetary variables

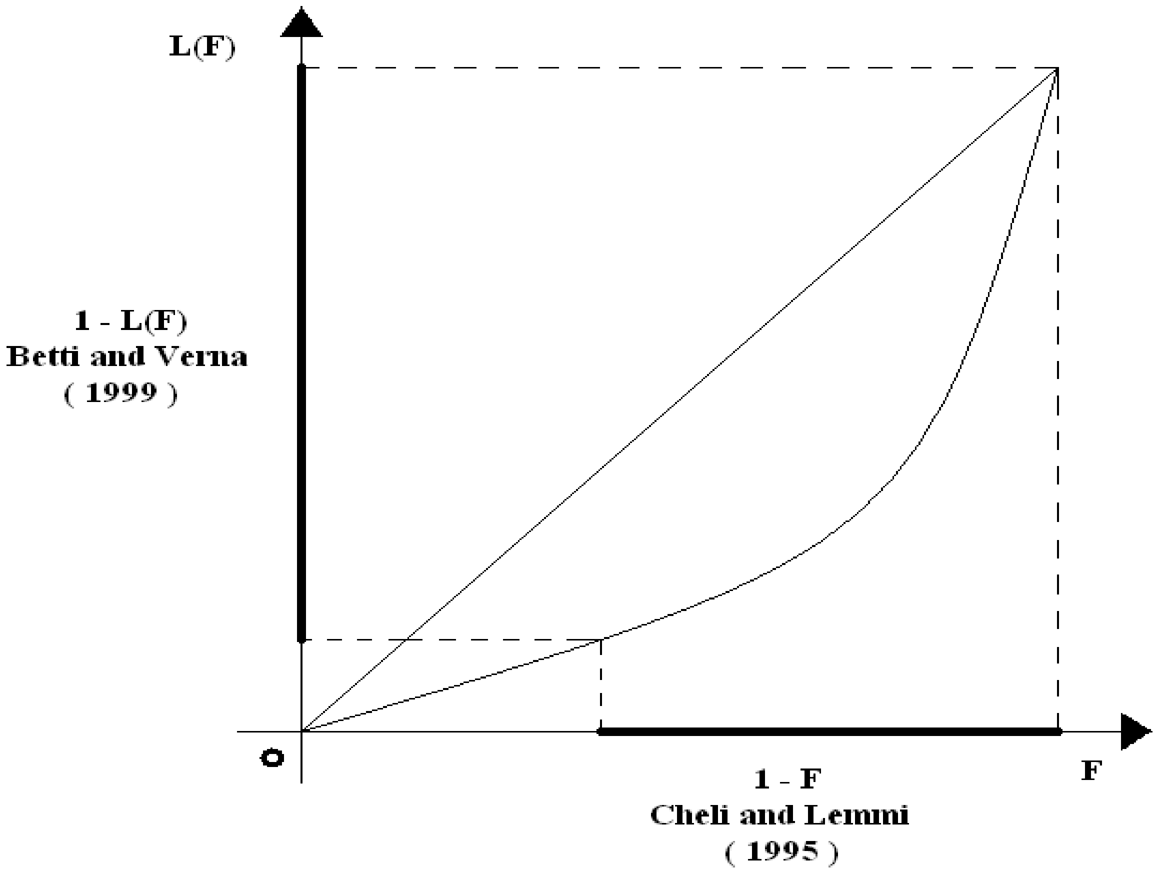
Cheli and Lemmi (1995)

$$\mu_i = \left(1 - F\right)^{\alpha} = \left(\frac{\sum_{l=i}^{n} w_i}{\sum_{l=1}^{n} w_i} \right)^{\alpha}$$

Betti and Verma (1999)

$$\mu_i = FM_i = \left(1 - L_{(M),i}\right)^{\alpha} = \left(\frac{\sum_{i=i+1} w_l y_l}{\sum_{l=1} w_l y_l} \right)^{\alpha}$$

Membership functions used by Cheli and Lemmi (1995), and Betti and Verma (1999)



Definition of the membership function based on monetary variables (diffusion)

Betti, Cheli Lemmi and Verma (2005, 2006)

$$\mu_i = FM_i = (1-F)^{\alpha-1}.[1-L(F)] = \left(\frac{\sum_{\gamma} w_{\gamma} \mid y_{\gamma} > y_i}{\sum_{\gamma} w_{\gamma} \mid y_{\gamma} > y_1} \right)^{\alpha-1} . \left(\frac{\sum_{\gamma} w_{\gamma} y_{\gamma} \mid y_{\gamma} > y_i}{\sum_{\gamma} w_{\gamma} y_{\gamma} \mid y_{\gamma} > y_1} \right)$$

Where parameter α is chosen so that the mean of the m.f. is equal to head count ratio H:

$$E(FM) = \frac{\alpha + G_{\alpha}}{\alpha.(\alpha+1)} = H$$

Non-monetary indicators: important aspects

- An index of non-monetary deprivation should be developed and analysed in its own right, separately from measures of income poverty.
- It is useful to combine the two types of measures in order to study the extent to which they overlap or are disjoint
- It is useful to decompose the overall index of non-monetary deprivation into underlying dimensions.
- An index of non-monetary deprivation should be supplemented by a set of indicators of deprivation in specific dimensions and aspects which are not suitable for incorporation into a single, overall index.
- In addition to indicators of individual-level deprivation, specifically *area-level* indicators should also be included.

Membership function based on supplementary variables (FS)

Variables and dimensions

Quantification and putting together a large set of non-monetary indicators of living conditions involves a number of steps, models and assumptions.

1. selection of indicators which are substantively meaningful and useful: mostly used ‘objective’ indicators
2. identifying underlying dimensions: this is done via factor analysis and sensible considerations; grouping the indicators accordingly
3. assigning numerical values to ordered categories
4. weighting of measures
5. scaling of measures

Membership function based on supplementary variables (FS) – diffusion

Here we have adopted the methodology of the *Second European report on Poverty, Income and Social Exclusion* (Eurostat, 2002)

Elementary indicators are combined to form an index describing an overall degree of deprivation. The individual's score averaged over items (k) is written as the weighted mean:

$$S_j = \Sigma_k \left(w_k . s_{j,k} \right) / \Sigma_k \, w_k$$

where the weights w_k are defined taking into account dispersion and correlation among items.

Calculation of weights

Weights comprise two factors: the dispersion of deprivation indicator and its correlation with other deprivation indicators in the given dimension

$$w_{hj} = w_{hj}^{a} \cdot w_{hj}^{b}, h = 1, 2, ..., m; j = 1, 2, ..., k_h$$

The first factor is the coefficient of variation of the complement to one of the deprivation scores *s* as follows:

$$w_{hj}^{a} \propto \frac{std_{hj}}{1 - mean_{hj}}$$

The second factor, as a measure of the correlation, can be computed in the following form:

$$w_{hj}^{b} \propto \left(\frac{1}{1 + \sum_{j=1}^{k_h} r_{e_{hj,hj'}} \mid r_{e_{hj,hj'}} < r_{e_{hj}}^{*}} \right) * \left(\frac{1}{1 + \sum_{j=1}^{k_h} r_{e_{hj,hj'}} \mid r_{e_{hj,hj'}} < r_{e_{hj}}^{*}} \right)$$

Non-monetary indicators: 5 dimensions

Dimensions and items of non-monetary deprivation

1 Basic non-monetary deprivation – these concern the lack of ability to afford most basic requirements:

Keeping the home (household's principal accommodation) adequately warm.
Paying for a week's annual holiday away from home.
Replacing any worn-out furniture.
Buying new, rather than second hand clothes.
Eating meat chicken or fish every second day, if the household wanted to.
Having friends or family for a drink or meal at least once a month.
Inability to meet payment of scheduled mortgage payments, utility bills or hire purchase instalments.

2 Secondary non-monetary deprivation – these concern enforced lack of widely desired:

A car or van.
A colour TV.
A video recorder.
A micro wave.
A dishwasher.
A telephone.

Non-monetary indicators: 5 dimensions

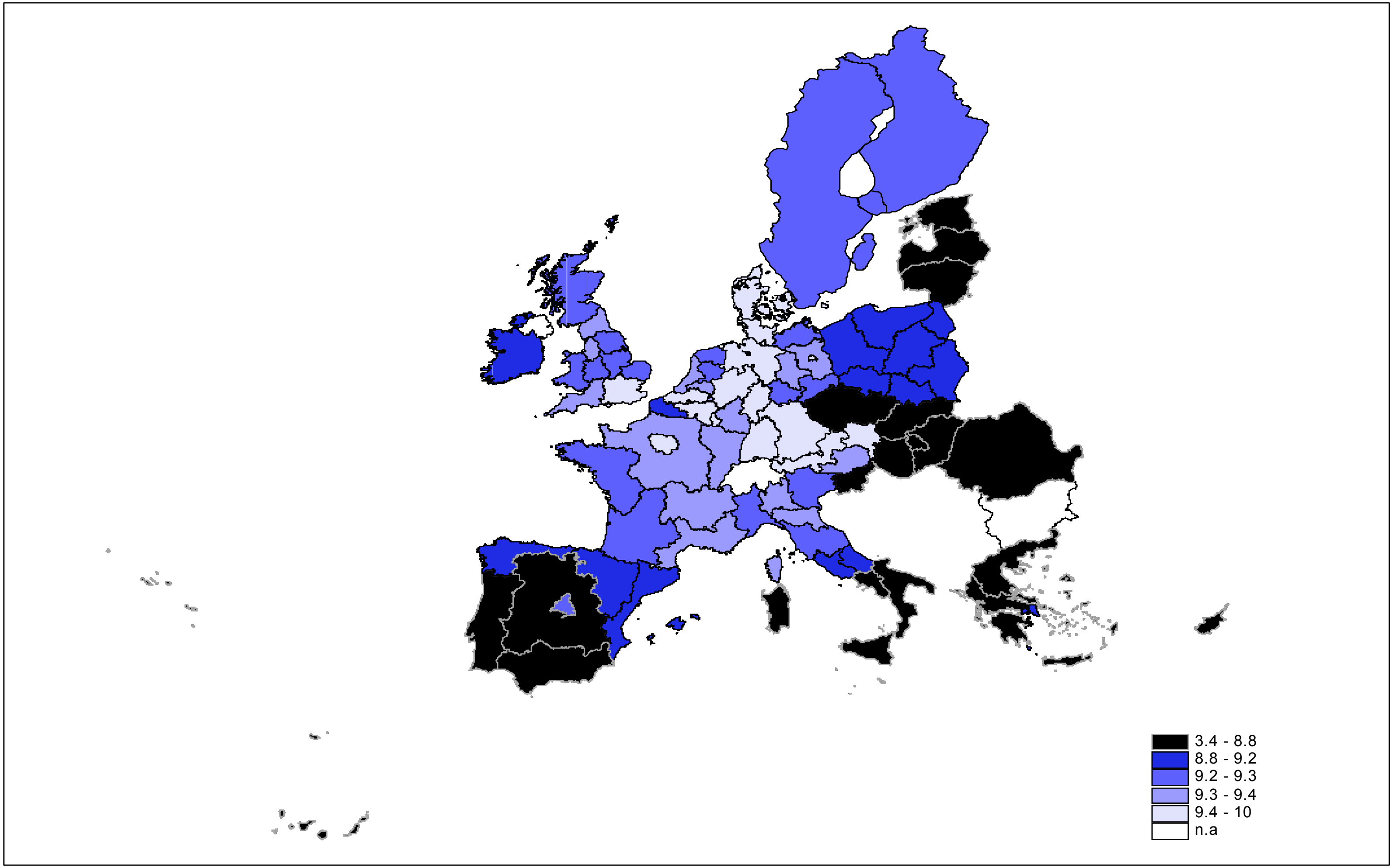
3 Lacking housing facilities – these concern the absence of basic housing facilities:
- A bath or shower.
- An indoor flushing toilet.
- Hot running water.

4 Housing deterioration – these concern serious problems with accommodation:
- Leaky roof.
- Damp walls, floors, foundation etc.
- Rot in window frames or floors.

5 Environmental problems – these concern problems with the neighbourhood and the environment:
- Shortage of space.
- Noise from neighbours or outside.
- Dwelling too dark/not enough light.
- Pollution, grime or other environmental problems caused by traffic or industry.
- Vandalism or crime in the area.

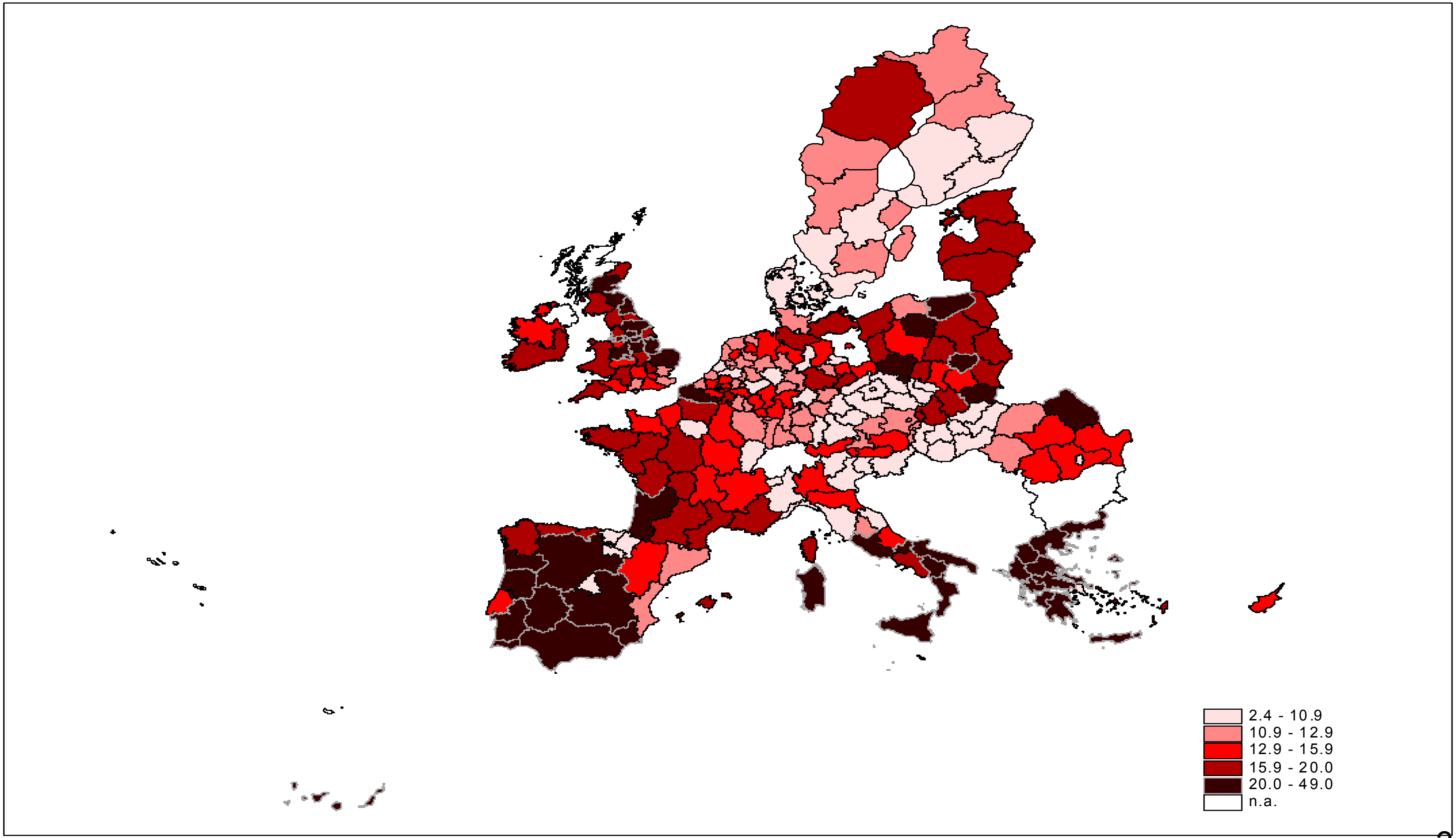
Some empirical results - 1

Figure 1. EU: Net equivalent income – NUTS1 regions



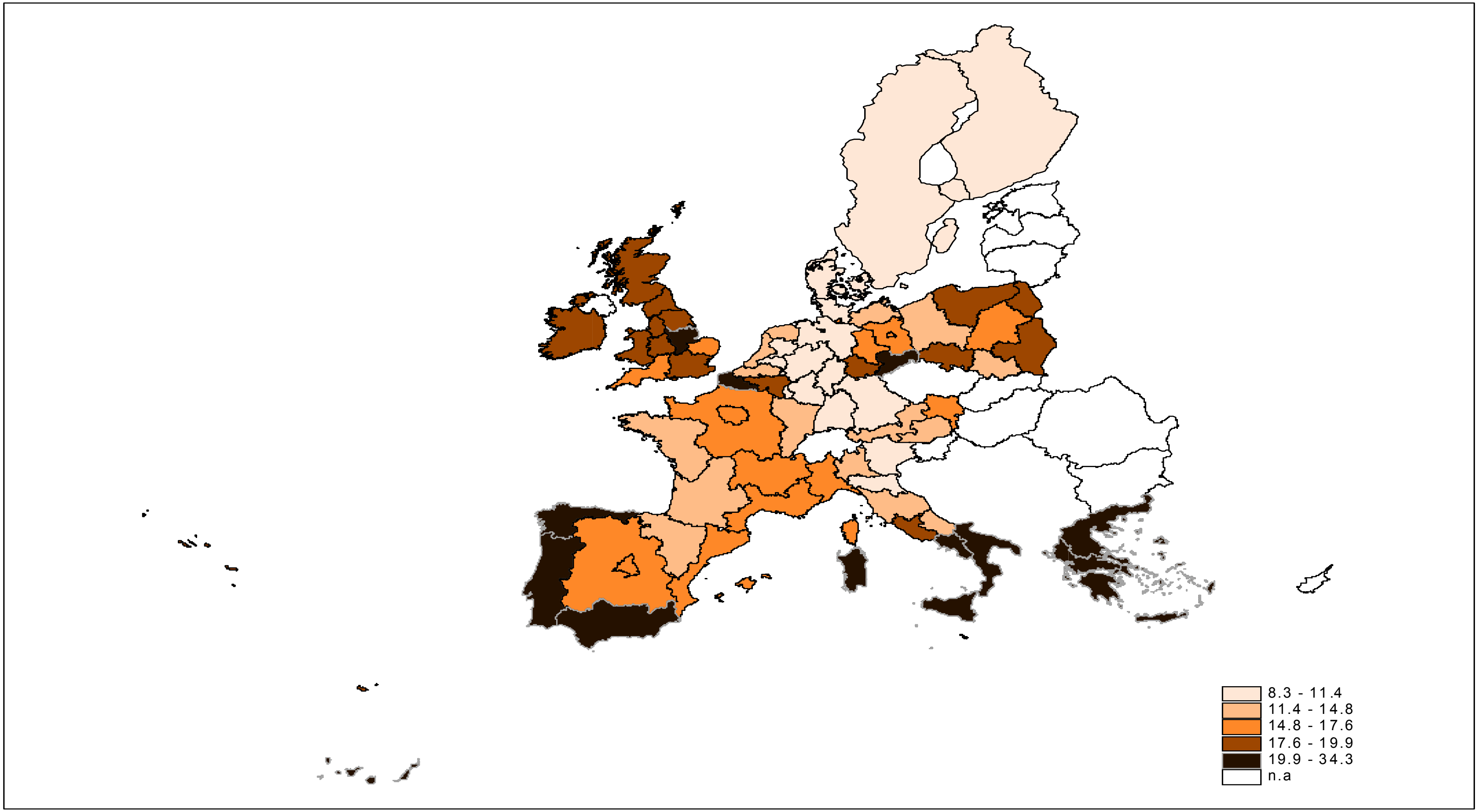
Some empirical results - 2

Figure 2. Head Count Ratio NUTS2 regions (country poverty lines)



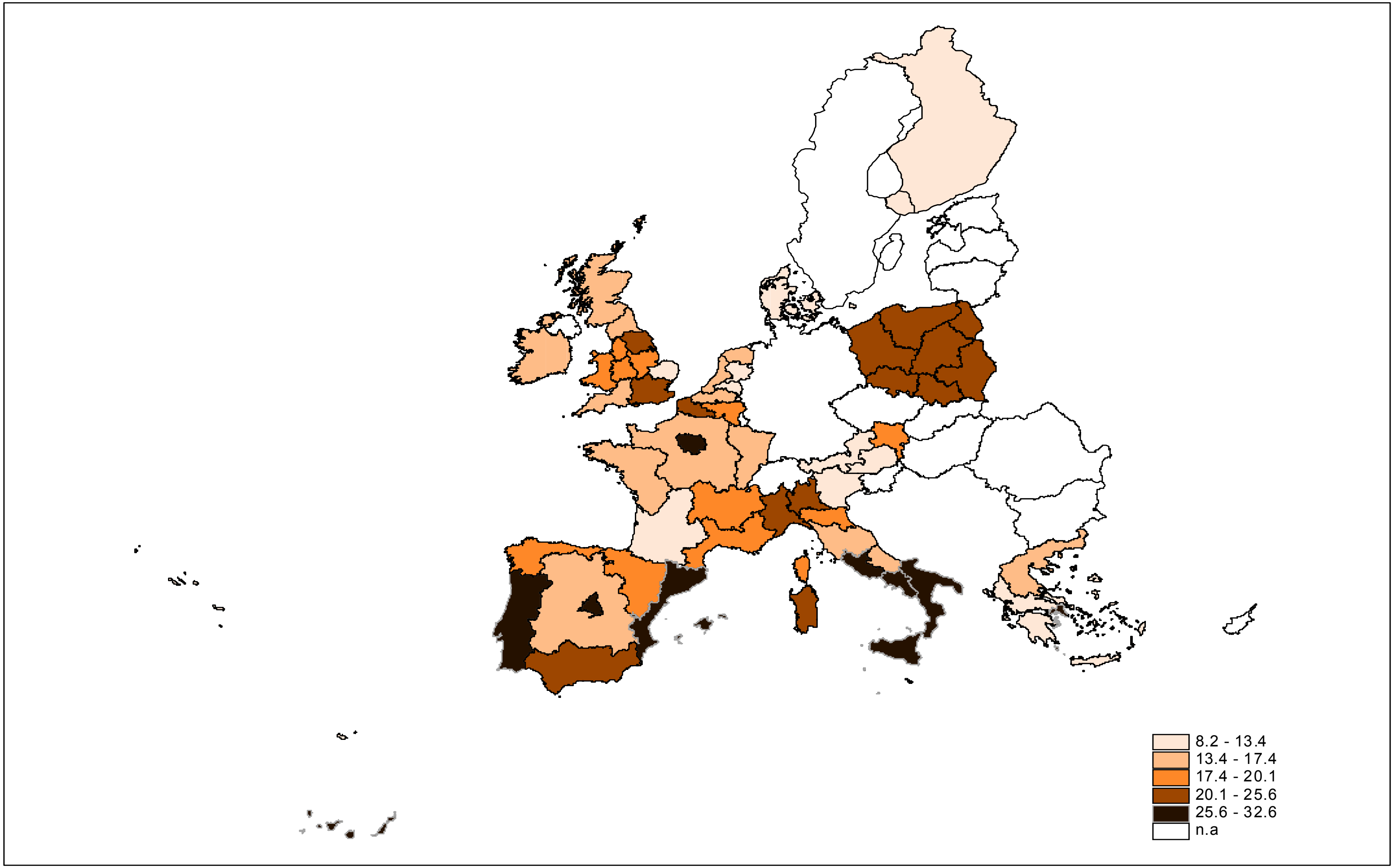
Some empirical results - 3

Figure 3. Overall Non-monetary deprivation rates, NUTS1 regions



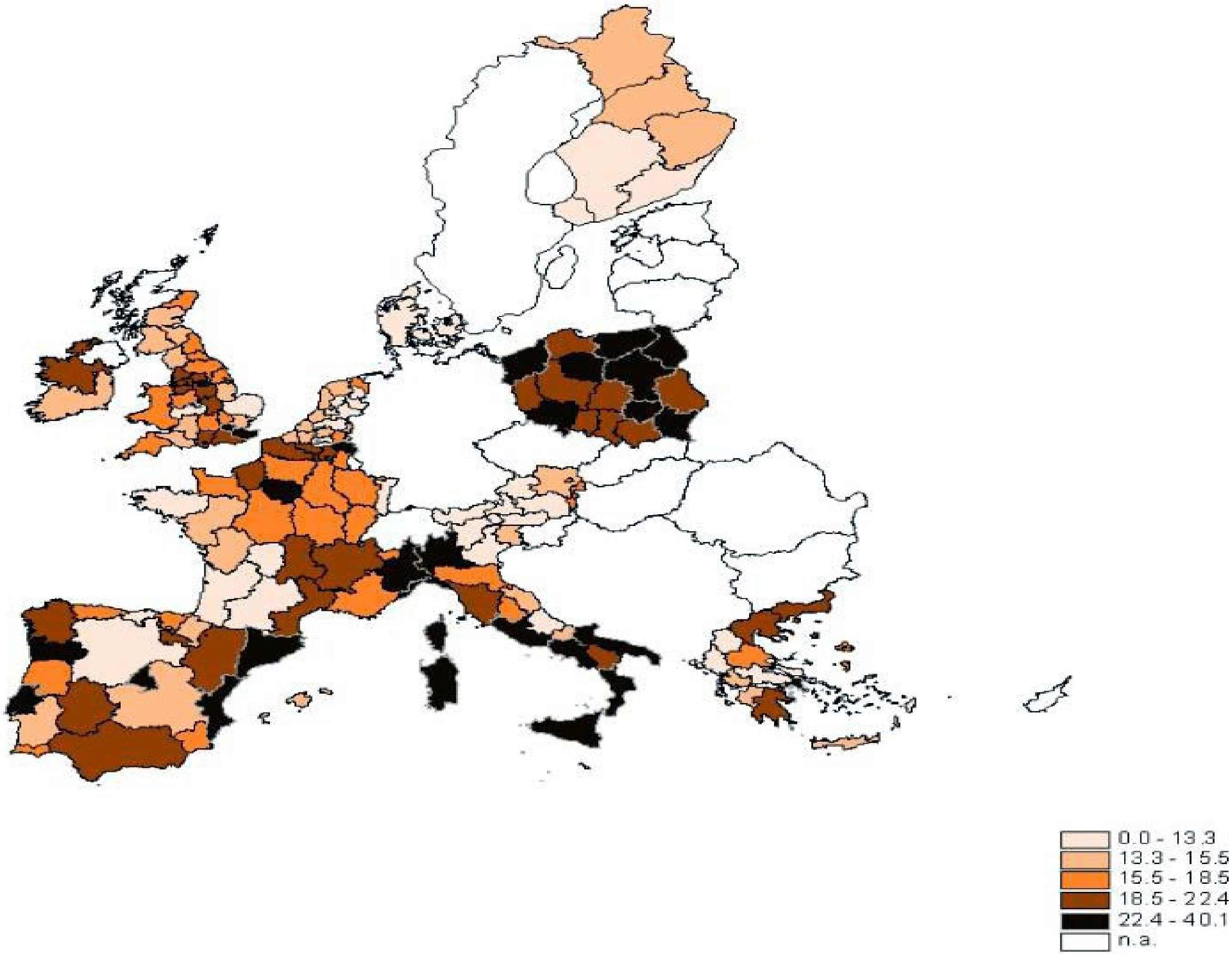
Some empirical results - 4

Figure 4. Environmental Problems, NUTS1 regions



Some empirical results - 5

Figure 5. Environmental Problems, NUTS2 regions



TFR Analysis of Poverty Dynamics:
Joint memberships over 2 periods

Let us indicate by $\mathbf{g}_{\mathbf{i}}^{(t)} = [g_{i0}^{(t)}, g_{i1}^{(t)}]$, $i = 1, \ldots, n$; $t = 1, \ldots, T$, the vector whose components represent the degrees of membership in the two fuzzy states that are coded 0 (absence of poverty) and 1 (presence of poverty). Subscript *i* refers to the statistical unit whereas superscript *t* refers to the panel wave.

Our scope is now defining the joint membership function over two consecutive periods as follows in the next slides

TFR Analysis of Poverty Dynamics:
Joint memberships over 2 periods

Time 1 \ Time 2	Non poor 0	Poor 1	Margins
Non poor 0	A $g_{i00}^{(1,2)}$	$g_{i01}^{(1,2)}$ B	$g_{i0}^{(1)}$
Poor 1	C $g_{i10}^{(1,2)}$	$g_{i01}^{(1,2)}$ D	$g_{i1}^{(1)}$
Margins	$g_{i0}^{(2)}$	$g_{i1}^{(2)}$	1

We define the joint m.f following Manton *et al* (1992):

$$g_{ikl}^{(1,2)} = \min[\, g_{ik}^{(1)} , g_{il}^{(2)} \,]$$

TFR Analysis of Poverty Dynamics: *Joint memberships over 2 periods*

Once one element of the 2 x 2 matrix has been chosen, the other three are determined by the marginal constrains.

There are therefore 4 possible matrix-solutions; as Manton *et al* (1992) suggested we consider the solution-matrix with minimum entropy, i.e.

$$H_i = -\sum_k \sum_l g_{ikl}^{(1,2)} \ln[g_{ikl}^{(1,2)}]$$

TFR Analysis of Poverty Dynamics: *Joint memberships over 2 periods*

Cheli and Betti (1999) demonstrated that the matrix-solutions "starting" from A and D coincide as well as those "starting" from B and C.

Moreover Betti, Cheli and Cambini (2004) proved that if the two propensity to poverty over the two periods are "concordant" (i.e. both less 0.5 or both higher 0.5) then the matrix with minimum entropy is the one starting from "A or D"; otherwise if they are "discordant" the matrix with minimum entropy is the one "starting" from "B or C".

TFR Analysis of Poverty Dynamics:
Joint memberships over 2 periods

Time 2

Time 1	Non poor 0	Poor 1	Margins
Non poor 0	$E\left(g_{i00}^{(1,2)}\right)$	$E\left(g_{i01}^{(1,2)}\right)$	$E\left(g_{i0}^{(1)}\right)$
Poor 1	$E\left(g_{i10}^{(1,2)}\right)$	$E\left(g_{i11}^{(1,2)}\right)$	$E\left(g_{i1}^{(1)}\right)$
Margins	$E\left(g_{i0}^{(2)}\right)$	$E\left(g_{i1}^{(2)}\right)$	1

Index of persistent poverty

Index of transitory poverty (blue)

Transition Matrix (without memory)

Following Manton *et al.* (1992) it is possible to define the transition matrix between fuzzy states:

$$t_{l|k}^{(1,2)} = \frac{\mathrm{E}[\, g_{ikl}^{(1,2)} \,]}{\mathrm{E}[\mathrm{g}_{ik}^{(1)}]}$$

We define it Transition matrix without memory.

Transition Matrix (with memory)

In this paper we introduce a first order memory in the Transition matrix as follows:

$$t_{m|kl}^{(1,2,3)} = \frac{E[g_{klm}^{(1,2,3)}]}{E[g_{kl}^{(1,2)}]}$$

where:

$$g_{iklm}^{(1,2,3)} = \min [\, g_{ikl}^{(1,2)} \;, g_{ilm}^{(2,3)} \quad g_{ikm}^{(1,3)} \,]$$

Dynamic Indices

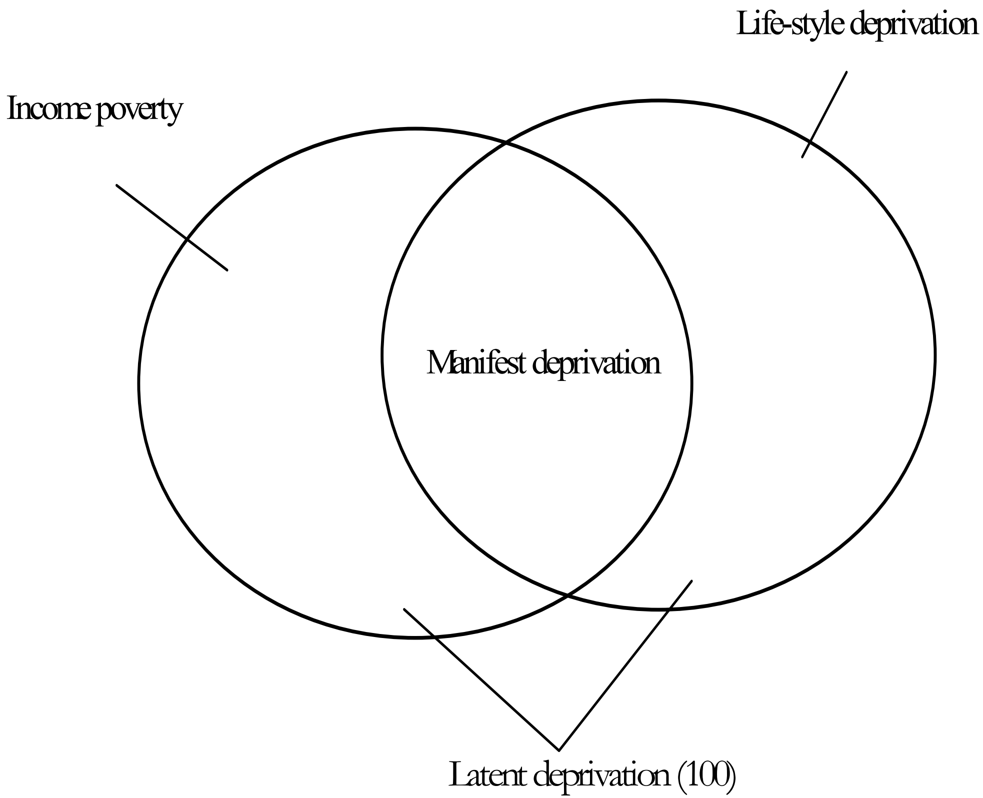
On the basis of the transition matrix with memory of the first order it is possible to define membership functions of any length:

$$\mathrm{E}[g_{k_1 k_2 \dots k_T}^{(1,2,\dots,T)}] = \mathrm{E}[g_{k_1 k_2 k_3}^{(1,2,3)}]\, t_{k_3 k_4 | k_2}^{(3,4|2)} \dots t_{k_{T-1} k_T | k_{T-2}}^{(T-1,T|T-2)}$$

And Dynamic indices of any length:

$$D^c = \sum \mathrm{E}[g_{k_1 k_2 \dots k_T}^{(1,2,\dots,T)}]$$

- **Income poverty and non-monetary deprivation in combination**



Application of the Composite set operations

The 'manifest' deprivation propensity of individual *j* is the intersection (the smaller) of the two measures FM_j and FS_j:

.

$$M_j = \min\left(FM_j,\ FS_j\right)$$

Similarly, the 'latent' deprivation propensity of individual *j* is the union (the larger) of the two measures FM_j and FS_j:

$$L_j = \max\left(FM_j,\ FS_j\right)$$

Longitudinal aspects: persistence of poverty and deprivation

Any-time poverty

The individual's propensity to 'any-time poverty' (i.e., poverty during at least one year over the interval) is given by the largest of the cross-sectional indices:

$$P_{(1),j} = \max\left(P_{t,j}\right)$$

Persistent poverty

We adopt the following definition of persistent poverty for the numerical results presented here. It refers to poverty during *at least a majority* of the T years, i.e. for at least T' years, where

T' = int(T/2) + 1 (i.e. the smallest integer strictly larger than T/2).

Continuous poverty

The individual's propensity to continuous poverty (i.e., for all the years over the interval) is the smallest of the cross-sectional indices:

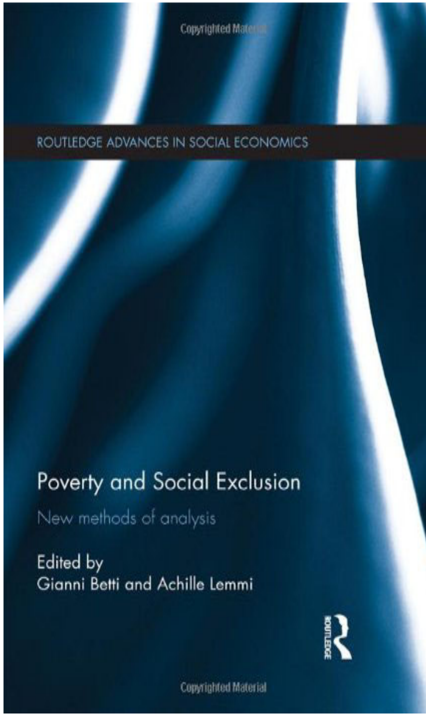
$$P_{(T),j} = \min\left(P_{t,j}\right)$$

References

Betti G., Gagliardi F., Lemmi A., Verma V. (2012), Sub-national indicators of poverty and deprivation in Europe: methodology and applications, *Cambridge Journal of Regions, Economy and Society*, 5(1), pp. 149-162.

Betti G., Lemmi A. (2013, eds.), Poverty and Social Exclusion: New Methods of Analysis. London and New York: Routledge.

European Commission (2005), Regional Indicators to reflect social exclusion and poverty. Report prepared for Employment and Social Affairs DG - with contribution of Gianni Betti, Achille Lemmi, Anna Mulas, Michela Natilli, Laura Neri, Nicola Salvati, and Vijay Verma.



THANK YOU FOR YOUR ATTENTION !!!