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Multivariate Small Area Estimation of Multidimensional Economic Well-being Indicators: a focus on Housing Quality

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Outline

1. Data dimensionality reduction

2. Small area estimation via multivariate EBLUP

3. Simulation study

4. Application based on EU-SILC data

5. Future work

Data dimensionality reduction

- Multivariate statistical analysis techniques
  - Depending on variable types: continuous, categorical, ordinal
- The problem of **weighting and aggregation** (OECD-JRC, 2008)
  - Big debate on the methodology
  - Statistical models taking into account for the **degree of correlation** among the observed variables (dashboard of indicators)
  - Different type of variables
- **Factor analysis models – the use of the factor scores**
  - Possible for any type of variables (transformations)
  - Good rankings, smaller variability compared to the traditional weighted averages
  - FA model variability needs to be take into account in MSE estimation (Moretti, Shlomo and Sakshaug, 2017a)

<u>The aim of this paper</u>

- Evaluate **EBLUP approach** under BHF model (Battese et al. 1988) and **<u>multivariate EBLUP</u>** (MEBLUP) under Fuller and Harter model (Fuller and Harter, 1987) in data dimensionality reduction for latent well-being indicators
- Extension of Moretti, Shlomo and Sakshaug (2017a): univariate latent well-being
- Case of observed indicator <u>a priori</u> selected and studied in well-being frameworks
- The case of **averages of standardised EBLUPs** compared to the EBLUP of **factor scores means (multivariate and univariate case)**
- Simulation study and application using European Union Statistics on Income and Living Conditions (EU-SILC) Italian data.

Multivariate EBLUP approach

Target parameter: a vector of K means for small area d:

$$\bar{\boldsymbol{Y}}_d = N_d^{-1}\sum_{i=1}^{N_d}\boldsymbol{y}_{di},$$

$$\bar{\boldsymbol{Y}}_d = N_d^{-1}\left(\sum_{i\in s_d}\boldsymbol{y}_{di} + \sum_{i\in r_d}\boldsymbol{y}_{di}\right)$$

Multivariate nested-error model (Fuller and Harter, 1987)

$$\boldsymbol{y}_{di} = \boldsymbol{x}_{di}\boldsymbol{\beta} + \boldsymbol{u}_d + \boldsymbol{e}_{di}, d = 1, \ldots, D, i = 1, \ldots, N_d,$$

$$\boldsymbol{u}_d \sim N_K(\boldsymbol{0}, \boldsymbol{\Sigma}_u), \qquad \boldsymbol{e}_{di} \sim N_K(\boldsymbol{0}, \boldsymbol{\Sigma}_e)\ \ \boldsymbol{u}_d \textit{ and } \boldsymbol{e}_{di} \textit{ are independent}$$

Model parameters estimated via ML (Datta et al. 1999)

Prediction (Fuller and Harter, 1987; Datta et al. 1999)

$$\hat{\bar{\boldsymbol{Y}}}_d^{MEBLUP} = \bar{\boldsymbol{X}}'_{dp}\hat{\boldsymbol{\beta}} + \hat{\boldsymbol{u}}_d = \bar{\boldsymbol{X}}'_{dp}\hat{\boldsymbol{\beta}} + (\bar{\boldsymbol{y}}_d - \bar{\boldsymbol{x}}'_d\hat{\boldsymbol{\beta}})[(\hat{\boldsymbol{\Sigma}}_u + n_d^{-1}\hat{\boldsymbol{\Sigma}}_e)^{-1}\hat{\boldsymbol{\Sigma}}_u], d = 1, \ldots, D$$

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Simulation study (1)

Generating the population

- From the multivariate nested-error model $\boldsymbol{y}_{di}$ with *k=1,…4* are generated with:

$$N = 20{,}000, \quad D = 80, \quad 130 \leq N_d \leq 420,$$

$$N_d \sim \mathcal{U}(a = 130, b = 420)$$

$$X_1 \sim \mathcal{U}(a = 145, b = 459)$$

$$X_2 \sim \mathcal{U}(a = 55, b = 345)$$

- Different correlation structure in $\boldsymbol{\Sigma}_e$ *and* $\boldsymbol{\Sigma}_u$:

  $r_u = r_e = 0.2,$

  $r_u = 0.2$ and $r_e = 0.7,$

  $r_u = -0.2$ and $r_e = 0.7$

- Intra-class correlation: $ICC_k = \{0.05,0.1,0.3\}$ so we generate $\boldsymbol{\Sigma}_u$ matrices as functions of *ICC*.

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Simulation study (2)

Simulation steps

1. Draw $S = 500$ samples using simple random sampling without replacement n=1000: allowing for **unplanned domains**

2. Estimate a *one-factor*, and *two-factors analysis* models and estimate the EBLUP and MEBLUP **factor score means** for each area d in each sample

3. EBLUP and MEBLUP on each of the original observed variables y and vectors $\boldsymbol{y}$ are also estimated: construct **simple averages** of the standardized small area EBLUPs and MEBLUPs' components and **weighted averages** using the factor loadings

4. Results are evaluated via RMSE, percentages of reduction in terms of RMSE, and relative bias.

Simulation study (3)

Case two-factor only

Table 1: Percentage relative reduction (%) in terms of RMSE of simple and weighted averages of standardised MEBLUP over EBLUP , two-factor CFA model .

		Scenario					
ICC_k		$r_e = 0.7, r_u = 0.2$		$r_e = 0.7, r_u = -0.2$		$r_e = 0.2, r_u = 0.2$	
		Simple	*Weighted*	*Simple*	*Weighted*	*Simple*	*Weighted*
0.05	*Factor 1*	-5.26	-5.29	-5.56	-6.52	-2.86	-2.94
	Factor 2	-3.90	-4.74	-20.99	-21.82	-0.43	-0.22
0.1	*Factor 1*	-8.89	-9.30	-26.67	-22.73	-0.50	-0.25
	Factor 2	-9.00	-11.08	-25.53	-26.67	0.00	0.00
0.3	*Factor 1*	-11.67	-11.88	-16.67	-17.92	-0.28	-0.29
	Factor 2	-13.89	-12.86	-19.87	-17.05	0.00	0.00

Simulation study (4)

Table 2: Percentage relative reduction (%) in terms of RMSE of MEBLUP over EBLUP, factor scores, two-factor CFA model

		Scenario		
ICC_k		$r_e = 0.7, r_u = 0.2$	$r_e = 0.7, r_u = -0.2$	$r_e = 0.2, r_u = 0.2$
0.05	Factor scores 1	-2.44	-2.50	0.00
	Factor scores 2	-2.50	-2.56	0.00
0.1	Factor scores 1	-2.56	-3.13	0.00
	Factor scores 2	-3.33	-2.86	0.00
0.3	Factor scores 1	-4.48	-5.56	0.00
	Factor scores 2	-5.56	-6.67	0.00

- The factor analysis model modified the correlation structure therefore percentage relative reductions change considerably

Application (1)

- Housing quality: dimension in multidimensional well-being Equitable and Sustainable Well-being **BES Italian framework**
- Determinant of well-being (Andrews et al. 2011)
- Data: **European Union Statistics on Income and Living Conditions** (EU-SILC) 2009 and **Census** of population and households 2001 (Censimento della popolazione e delle abitazioni)
- Small areas: Tuscany municipalities, unplanned domains in EU-SILC

Application (2)

Table 3: Factor loadings for two latent factors using explanatory factor analysis on EU-SILC observed variables

Variable	Factor 1	Factor 2
Severe material deprivation	0.010	**0.733**
Smog	**0.757**	0.025
Noise	**0.617**	0.154
Crime	**0.659**	0.130
Housing ownership	0.096	**-0.589**
Presence of humidity	0.010	**0.596**
Darkness inside the house	-0.002	**0.551**
Absence of rubbish in the street	**-0.843**	0.084
Absence of damages in public buildings	**-0.810**	0.012
Log equivalised disposable income	0.139	**-0.398**

Factor 1: residential area deprivation

Factor 2: housing material deprivation

Goodness-of-fit two-factor

RMSEA: 0.040

CFI: 0.925

TLI: 0.901

Data dimensionality reduction

Factor scores: FA allowing for different types of observed variables

No multivariate EBLUP on the dashboard: variables are on different scales

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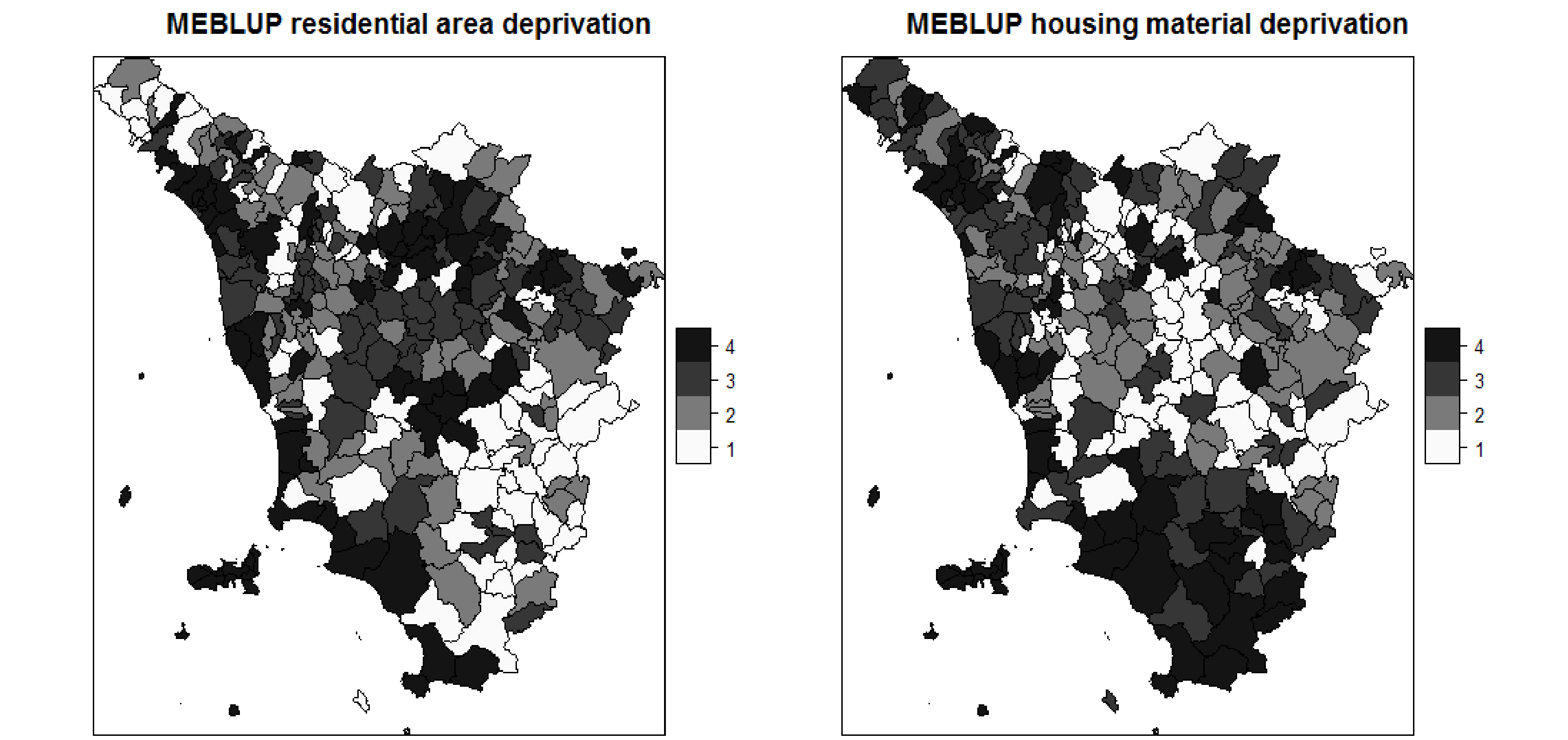
Application (3)



Percentile	0%	25%	50%	75%	100%
Residential area deprivation	0.000	0.261	0.266	0.270	1.000
Housing material deprivation	0.000	0.418	0.457	0.502	1.000

- **MSE** estimated via parametric bootstrap (Moretti, Shlomo and Sakshaug, 2018**)**
- **Percentage relative reduction (averaged)**

  6.41% and 7.90%

$$\hat{r}_e = 0.10, \hat{r}_u = 0.78$$

$$\widehat{ICC}_{f_1} = 0.21, \widehat{ICC}_{f_2} = 0.0.09$$

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Conclusion and future work

- Confirmed what found previously in the multivariate SAE literature: **gains in efficiency** of multivariate EBLUP over the univariate EBLUP depend on *ICC*, sign and magnitude of r_u and r_e
- Since factor analysis models may change the **correlation (and variance) structure in the observed data**, this needs to be considered in the multivariate modelling: multivariate EBLUP may not provide large gains
- **Current and future work** is about the use of multivariate generalised mixed models in small area estimation: allowing for different types of responses
- Factor analysis models allow for different types of responses in data dimensionality reduction.

Thank you

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